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Interim Report

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LIST OF SYMBOLS

d = still water depth

H = wave height

L = wave length

t = time

T = wave period

v = horizontal component of particle velocity

x = horizontal displacement of a particle

z = mean orbit depth below still water

I. INTRODUCTION

The forces experienced by a body fixed in an unsteady fluid flow are of two kinds; (1) drag force, which is due to the viscous nature of the fluid, with magnitude proportional to the square of the fluid velocity, and (2) inertia force, which is a consequence of Newton's Second Law, with magnitude proportional to the fluid acceleration. Wave motion is a particular case of unsteady flow in which the velocity and acceleration vary periodically with time, hence a theoretical expression for the force on a body, such as a pile, in the presence of wave motion may be written:

$$F = F_D + F_i = \frac{1}{2} C_D \rho A \left[v(t)\right]^2 + C_M \rho V \left[a(t)\right]$$

where C_D and C_M are the drag and inertia (or mass) coefficients respectively, ρ is the mass density of the fluid, A is the cross-sectional area, and V the volume of the body, and the velocity and acceleration are written as functions of time to emphasize the time-dependency of the total force.

In order to determine the maximum force on the body, therefore, it is necessary to maximize the entire expression with respect to time, and it will be recognized that this requires a detailed knowledge of $v(t)$ and $a(t)$.

In recent years, investigators at the University of California have made much progress in the development of rational methods for computing the wave forces on structures of engineering significance by experimentally determining values of C_D and C_M. In this work, however, the assumption was made that the time-dependent functions, a and v, could be closely approximated by various theoretical expressions.

It was the purpose of this investigation, therefore, to apply precise methods of measurement to the correlation of experimentally-determined velocity and acceleration time histories with accepted theoretical values. It was not expected that major differences would be discovered, but in view of the complicated nature of the expression for total force, it is apparent that even small differences of amplitude or phase could result in appreciably different maximum forces.

A further consideration in the undertaking of this interim project was the availability of the major equipment required (channel, stroboscopic lighting, photographic) and personnel while waiting for the fabrication of the wave force balance.

II. EXPERIMENTAL TECHNIQUE

General

The particle motion studies were performed in the 3-ft deep, 2-ft wide and 100-ft long laboratory wave channel. The wave generator in this channel is of the inverted ballistic-pendulum type which is capable of imparting particle motion to the water which is of the proper horizontal dimensions from surface to bottom. To absorb the wave energy and to prevent unwanted reflections, a gravel beach was installed with a slope of fifteen to one at the west end of the channel. A light-tight shed (Fig. 1) was constructed over the channel to house the photographic equipment used in taking the trajectory photographs. This shed was about 45 ft from the wave machine where there is 1/2-in. plate glass on both sides of the channel. For convenience, the shed had access from both sides of the channel. The portions of the channel outside the shed were covered with 15-lb building felt to exclude stray light.

Globules

Particle motion may be photographed by placing in the water an opaque globule of material of the same density as the water. This globule must be large enough and white enough to be readily discernible on a photograph and yet small enough compared with orbit dimensions to behave as a molecule rather than as a solid body. For the purpose of this study a particle diameter of 1/16 in. was determined to be the desirable size.

The globule material found most satisfactory was a mixture of Alkazene 47* (sp. gr. 1.294) and heptane (sp. gr. 0.684) with zinc oxide added for whiteness. The specific gravity of the mixture was regulated by adding a few drops of heptane to lighten it or Alkazene 47 to make it heavier. Droplets of this mixture were discharged into the center plane of the channel by means of a hypodermic syringe (Fig. 2) which had a long, fine metal tube with a tip of 0.024-in. internal diameter. The vertical position of the tip was adjustable, thus permitting a complete range of particle depths.

The procedure used to release a single particle into the water was to push enough fluid through the tip to form the desired size of particle,

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*A Dow Chemical Company product.

and then to simply pull the tip away leaving the globule at its initial depth. Figure 3 shows a globule on the tip just prior to being released.

Photographic

To record the orbital motion of the globule, multiple exposure photographs using stroboscopic lights were taken at rates of twenty and thirty exposures per second. A 4 x 5 view camera was securely fastened to a special fixed mount and the photographs were made through the glass wall of the channel. The focal distance from camera to the center plane of the channel was 4 ft. The procedure in making an exposure was to open the shutter for one complete wave cycle while the stroboscopic lights were flashing at a predetermined fixed rate. The timing of one wave cycle was made convenient by a special timing light connected to the wave machine. The stroboscopic lights were mounted in 12-in. long tubes which beamed the light and prevented stray light from reaching the film. The lights were placed above the channel to give overhead illumination of the globules.

The stroboscopic lighting equipment was loaned by the Hydrodynamics Laboratory of the Institute campus where it had been developed for cavitation research, projectile sinking rate studies, etc. The equipment employs a 4000-volt power supply and thyratron pulser circuit to furnish an input of 2 watt-seconds of energy per flash per lamp. The lamps are General Electric FT-125 zenon-filled tubes, and the flash duration is 3 to 4 microseconds.

The grid (Fig. 3) which was used on all the photographs was obtained by double exposure. A special frame supported the grid in the central plane of the channel. Illumination was from the back and was obtained by the use of two photofloods. To minimize the number of installations of the grid, several films were exposed with each operation. After these exposures, the grid was removed, thus clearing the way for the orbit globules which were placed in the same plane as the grid had previously occupied. The grid was made by etching a 20- x 24-in. exposed and developed photographic plate on the emulsion side. Clear lacquer protected the emulsion while the grid was in the water.

Wave Height Measurements

The wave heights corresponding to each trajectory photograph were obtained by the use of two submergence elements placed upstream and

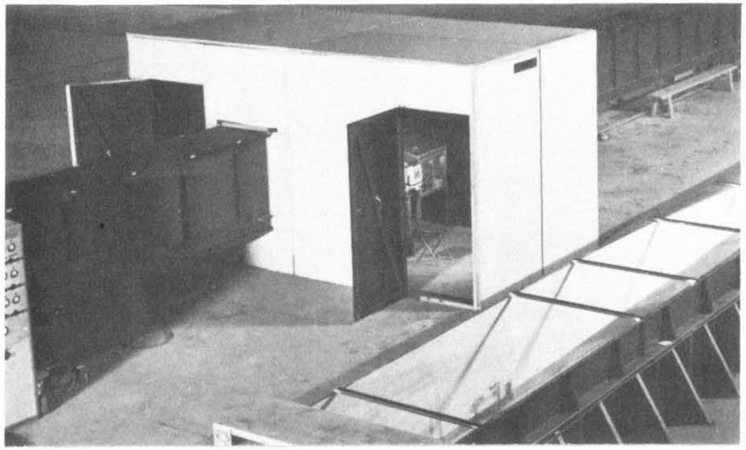

Fig. 1 - Darkroom for trajectory photographs

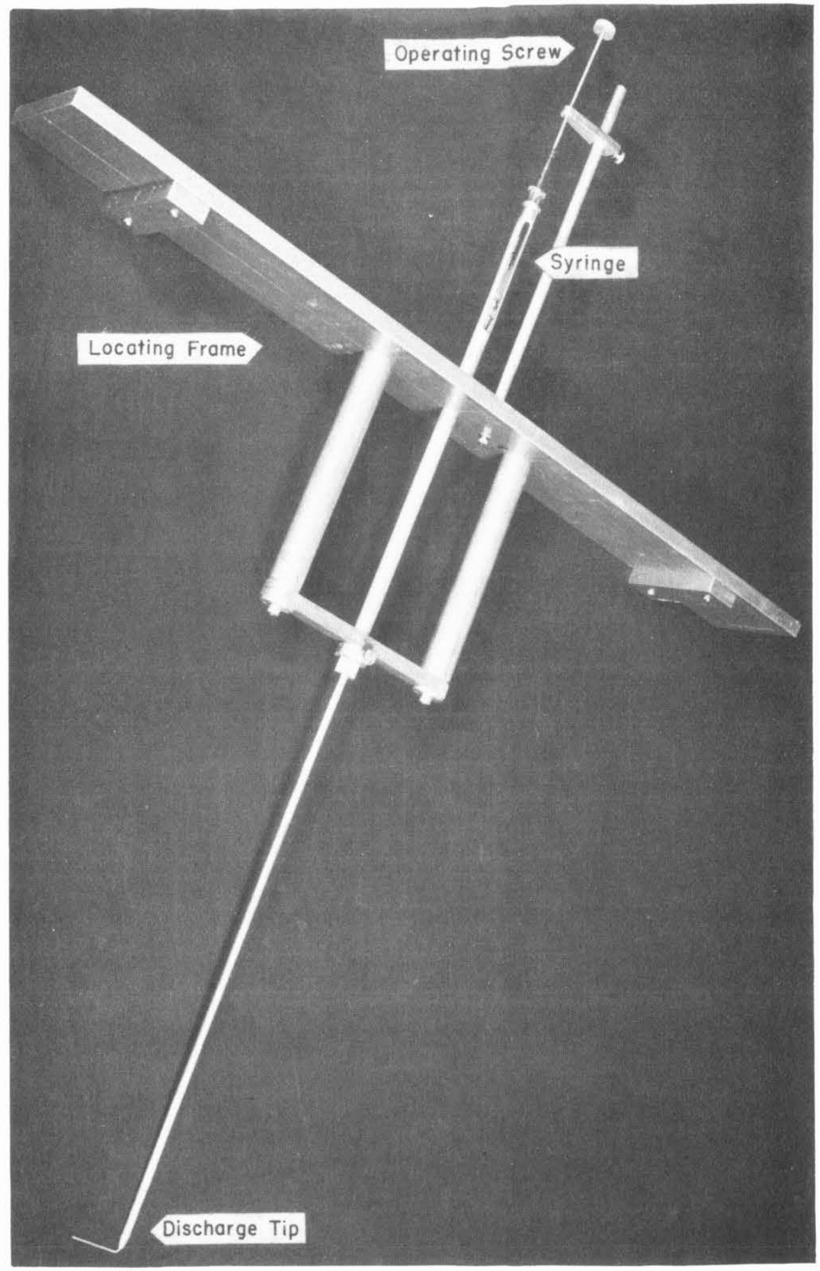

Fig. 2 - Globule injection assembly

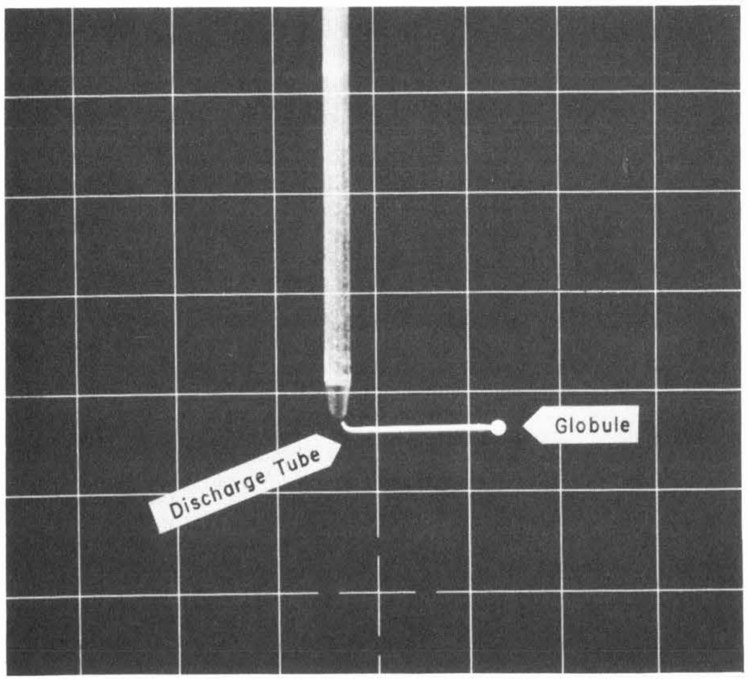

Fig. 3 - Globule prior to release

downstream from the test section of the channel. The wave heights were recorded as usual on an oscillograph, a manual coding device correlating the wave records with the trajectory photographs.

III. TEST PROGRAM

Since the investigation was an interim project, time limitations prevented a complete investigation of all wave parameters and orbit depths, so only three typical wave conditions were tested. These were the wave lengths of 8.4, 14.3, and 24.7 ft with corresponding heights of 3.56, 5.42, and 5.80 in., respectively, the water depth in all cases being 24.0 in. Seven orbit depths were observed for each wave length, so that a total of 21 photographs was necessary for the entire series of runs. To obtain 21 sharply focused photographs, however, it was necessary to take over 50 photographs in all. This was due mainly to the fact that it was difficult to make the globule stay within the very small depth of field of the camera lens. Best results were obtained by releasing the globule in still water, starting the wave machine, and then making the one-cycle exposure immediately after the wave became stable.

The small depth of field of the lens was an advantage in determining whether the globule was in the plane of the reference grid. At 4-ft focal distance, the depth of field of the 8-in. Ektar lens was one-half inch, and since only globules within this range were analyzed, the error of horizontal displacements was less than one percent. Because this is the main source of error in obtaining the trajectory photographs, it is believed that the experimental technique as described in this report is a valuable contribution to experimental wave research.

IV. ANALYSES OF TRAJECTORY PHOTOGRAPHS

Figure 4 shows typical trajectory photographs. Since the time between the "dots" is equal, the change in the horizontal component of velocity is readily apparent due to the change in the horizontal spacing of the dots. To measure the horizontal spacing accurately, the original negative was placed on a comparator and viewed through a microscope; by reading tangents to the dots, horizontal measurements could be obtained to ± 0.0003 in. on the film. For a typical measurement this error amounts to one-half of one percent. With the dimensions of the grid known, the absolute globule displacement in the channel was computed quite accurately. Similarly, the mean depth of the orbit could be obtained simply by noting its position on the grid.

The displacement of the globules can be expressed as:

$$x = x_1, x_2, x_3, \ldots . x_n,$$

where x_1, etc., are displacements from a datum line, the velocity as:

$$v = \frac{dx}{dt} = \frac{(x_{1,} - x_2)}{\Delta t}, \quad \frac{(x_2 - x_3)}{\Delta t}, \ldots .$$

where Δt is the time between light flashes; and the acceleration as:

$$\frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{(x_1 - x_2) - (x_2 - x_3)}{\Delta t^2}, \ldots .$$

By taking first and second differences as indicated above, velocities and accelerations were computed and the results are discussed in the next section.

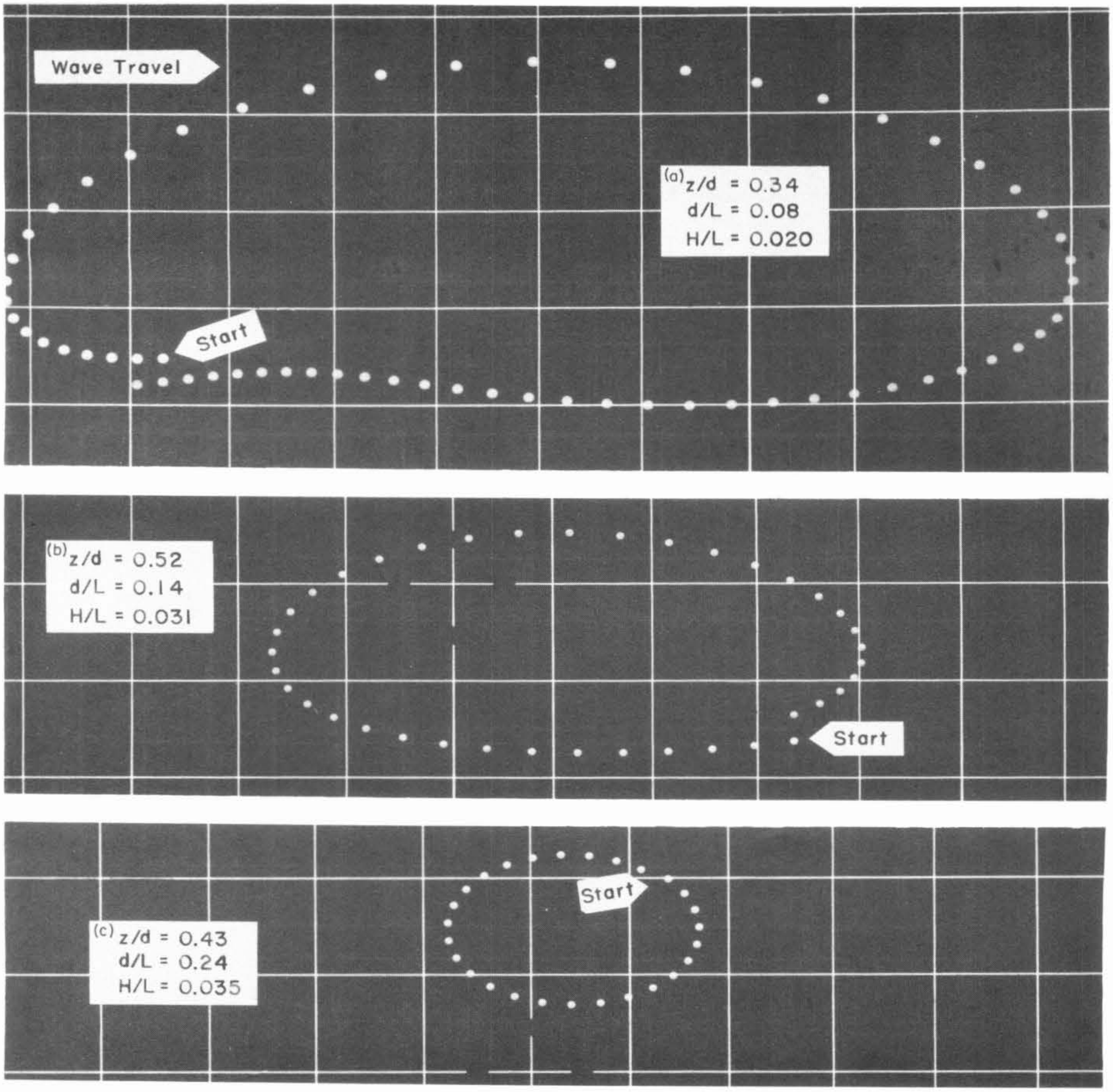

Fig. 4 - Typical trajectory photographs

V. RESULTS AND DISCUSSION

Velocities and accelerations as functions of time were computed for each of the 21 test runs. Figure 5 shows typical plots for each of the three wave lengths tested, the comparison curves being those given by the Stokes irrotational theory. Approximate equations for the velocity and acceleration as given by this theory[1, 2, 3] are:

$$\frac{dx}{dt} = -\frac{\pi H}{T}\,\frac{\cosh\frac{2\pi(d+z)}{L}}{\sinh\frac{2\pi d}{L}}\cos\frac{2\pi t}{T}$$

$$+\frac{(\pi H)^2}{LT\left(\sinh\frac{2\pi d}{L}\right)^2}\left[-\frac{1}{2}+\frac{3}{4}\,\frac{\cosh\frac{4\pi(d+z)}{L}}{\left(\sinh\frac{2\pi d}{L}\right)^2}\right]\cos\frac{4\pi t}{T}$$

$$+\frac{(\pi H)^2}{2LT}\,\frac{\cosh\frac{4\pi(d+z)}{L}}{\left(\sinh\frac{2\pi d}{L}\right)^2}$$

$$\frac{d^2x}{dt^2} = +\frac{2\pi^2 H}{T^2}\,\frac{\cosh\frac{2\pi(d+z)}{L}}{\sinh\frac{2\pi d}{L}}\sin\frac{2\pi t}{T}$$

$$-\frac{4\pi^3 H^2}{LT^2\left(\sinh\frac{2\pi d}{L}\right)^2}\left[-\frac{1}{2}+\frac{3}{4}\,\frac{\cosh\frac{4\pi(d+z)}{L}}{\left(\sinh\frac{2\pi d}{L}\right)^2}\right]\sin\frac{4\pi t}{T}$$

The point where $t/T = 0.50$ was used as the common point in drawing the experimental and theoretical curves.

A plot of the maximum positive and negative accelerations and velocities against orbit depth ratio (Fig. 6) shows very good agreement between the experiment and the theory of Stokes. As was expected, the agreement was best in the case of the shortest wave length (Fig. 6a). For this wave length the Stokes equations reduce to nearly a sine and cosine

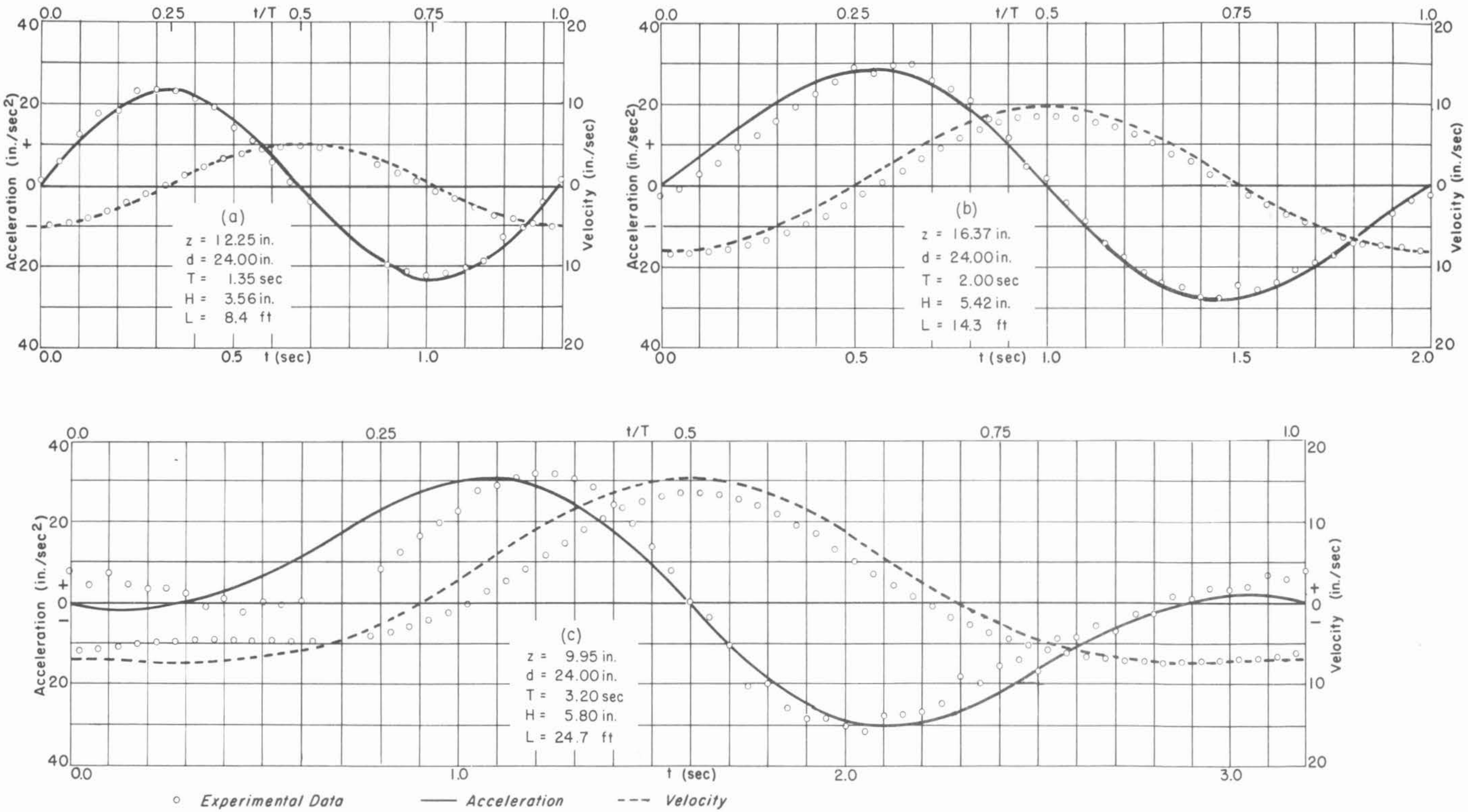

Fig. 5 - Comparison of horizontal velocity and acceleration with Stokes theory

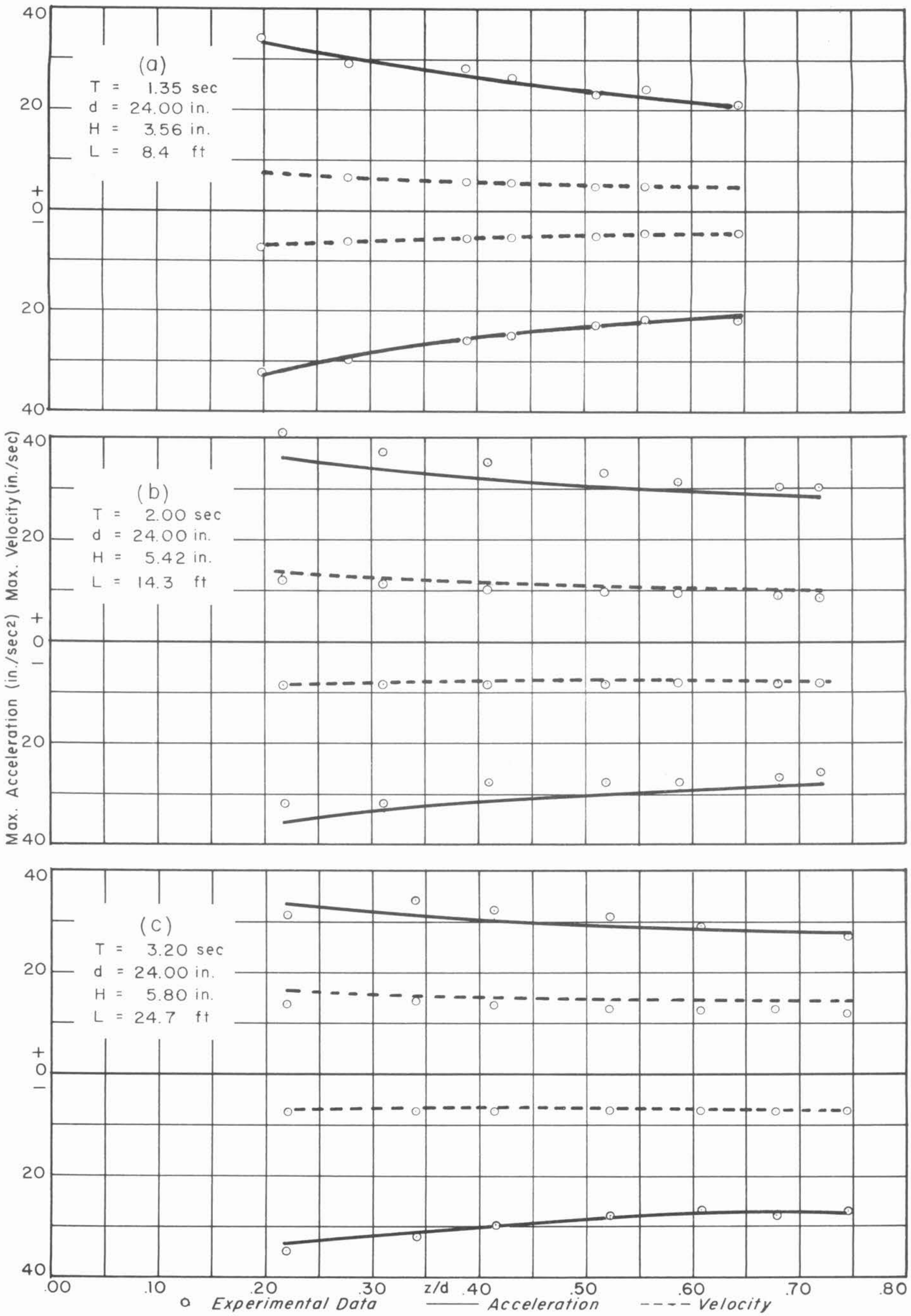

Fig. 6 - Comparison of maximum horizontal velocity and acceleration with Stokes theory

function (Fig. 5a).

While the agreement of the maximum value was good in all cases, the position of the maximum acceleration deviated by about 20 degrees from the theoretical for the 24.7-ft wave (Fig. 5c). This difference occurred at both the plus and minus maxima and was apparent at all the orbit depths of this wave length. It should be noted that the second acceleration maximum (Fig. 5c) for the 24.7-ft wave was very apparent in the experimental data.

Figure 4 shows that the orbits of the globules do not close in the vertical or horizontal direction. The direction of horizontal nonclosure is opposite to that expected by mass transport and, therefore, must be attributed to "closed" channel effects. Since the horizontal nonclosure is small compared with the total orbit diameter, the gap was neglected in the results reported here. Any vertical nonclosure of the orbit must be attributed to a globule which is rising or falling due to incorrect density. This vertical nonclosure, however, will have little effect on the horizontal displacements except as it may slightly affect the orbit depth. For this reason, the vertical gap was also neglected in analyzing the photographs.

VI. CONCLUSIONS

An approximation of the Stokes wave theory was compared for several wave conditions. Keeping in mind the limited nature of the investigation, the following conclusions seem justified:

1. The theoretical and experimental horizontal particle velocities and accelerations show very good agreement for waves of $d/L > 0.14$ and $H/L < 0.035$.

2. For waves approaching $d/L = 0.08$, a shift can be expected in the location of the maximum acceleration although the value of this maximum may still agree with the theory.

3. Maximum values of horizontal velocity show agreement down to at least $d/L = 0.08$ without any appreciable shift in location.

In summary, it may be stated that the Stokes equations should give good values for horizontal particle velocity and acceleration down to at least $d/L = 0.14$ for waves of ordinary steepness. As expected, best agreement between theory and measurement occurs with the shorter waves. Since, however, the tests described begin to show disagreement between theory and experiment for the longer waves, it appears that future investigations should be undertaken to test the longer waves more thoroughly. The technique described in this report might also be applied to the investigation of waves of large steepness near the breaking point, where accelerations are high and available theories are more in question.

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